Hoofdstuk 2: Oppervlakte en inhoud.

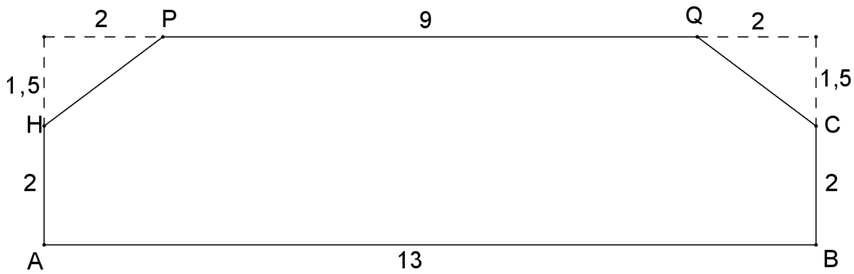
2.1 Oppervlakte van vlakke figuren

Opgave 1:

De oppervlakte van de figuur is precies de oppervlakte van een rechthoek van 7 bij 3 , dus $Opp = 7 \cdot 3 = 21$

Opgave 2:

a.



$Opp(ABCQPH) = 13 \cdot 3\frac{1}{2} - 2 \cdot \frac{1}{2} \cdot 2 \cdot 1\frac{1}{2} = 42\frac{1}{2}$

dus lijnstuk PQ verdeelt de achthoek niet in twee stukken met gelijke oppervlakte

b. $Opp(\Delta AIE) = \frac{1}{2} \cdot 9 \cdot 9 = 40\frac{1}{2}$

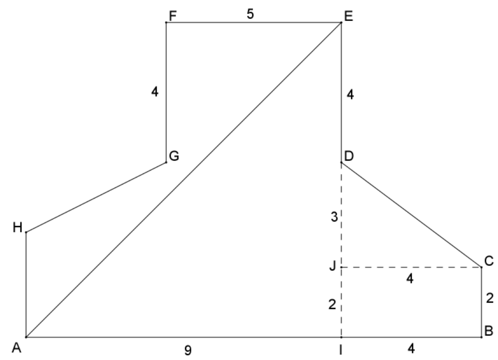
$Opp(BCJI) = 4 \cdot 2 = 8$

$Opp(\Delta CDJ) = \frac{1}{2} \cdot 4 \cdot 3 = 6$

$Opp(ABCDE) = 40\frac{1}{2} + 8 + 6 = 54\frac{1}{2}$

$Opp(AEFH) = 73 - 54\frac{1}{2} = 18\frac{1}{2}$

$Opp(I) : Opp(II) = 54\frac{1}{2} : 18\frac{1}{2} = 109 : 37$



Opgave 3:

a. $Opp(ABCD) = \frac{1}{2} \cdot Opp(AEFD) = \frac{1}{2}(a+b) \cdot h$

b. $Opp(ABCD) = Opp(\Delta ABD) + Opp(\Delta BCD) = \frac{1}{2} \cdot a \cdot h + \frac{1}{2} \cdot b \cdot h = \frac{1}{2}h(a+b)$

Opgave 4:

$Opp(figuur\ a) = \frac{1}{2} \cdot 4 \cdot 2 = 4\,\text{cm}^2$

$Opp(figuur\ b) = 4 \cdot 3 - \frac{1}{2} \cdot 3 \cdot 1 - \frac{1}{2} \cdot 3 \cdot 2 - \frac{1}{2} \cdot 4 \cdot 1 = 5\frac{1}{2}\,\text{cm}^2$

$Opp(figuur\ c) = Opp(parallel\log ram) + Opp(cirkel) = 4 \cdot 3 + \pi \cdot 1^2 = 12 + \pi = 15{,}14\,\text{cm}^2$

$Opp(figuur\ d) = Opp(trapezium) = \frac{1}{2} \cdot 3 \cdot (5+1) = 9\,\text{cm}^2$

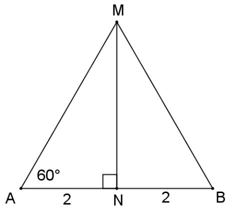
$Opp(figuur\ e) = Opp(rechthoek) + Opp(halve\ cirkel) = 3 \cdot 2 + \frac{1}{2} \cdot \pi \cdot 1^2 = 6 + \frac{1}{2}\pi = 7{,}57\,\text{cm}^2$

Opgave 5:

$AB = \sqrt{AM^2 + BM^2} = \sqrt{3^2 + 3^2} = \sqrt{18}$

$Opp = Opp(cirkel) - Opp(vierkant) = \pi \cdot 3^2 - (\sqrt{18})^2 = 9\pi - 18 = 10{,}27$

Opgave 6:



a. $\angle M = 360° : 6 = 60°$

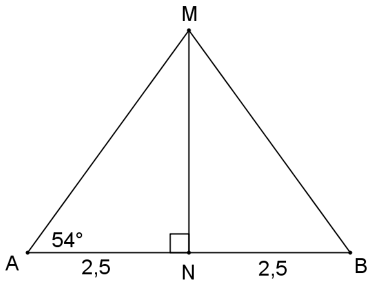
b. $\tan 60° = \dfrac{MN}{AN} = \dfrac{MN}{2}$

$MN = 2 \cdot \tan 60° = 3{,}464$

$Opp(\Delta ABM) = \tfrac{1}{2} \cdot 4 \cdot 3{,}464 = 6{,}93$

c. $Opp(ABCDEF) = 6 \cdot Opp(\Delta ABM) = 6 \cdot 6{,}93 = 41{,}6$

Opgave 7:



In ΔABM geldt: $\angle M = 360° : 5 = 72°$

$\angle A = \angle B = (180 - 72) : 2 = 54°$

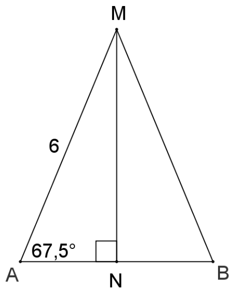
$\tan 54° = \dfrac{MN}{AN} = \dfrac{MN}{2{,}5}$

$MN = 2{,}5 \cdot \tan 54° = 3{,}44$

$Opp(\Delta ABM) = \tfrac{1}{2} \cdot 5 \cdot 3{,}44 = 8{,}60$

$Opp(ABCDE) = 5 \cdot Opp(\Delta ABM) = 5 \cdot 8{,}60 = 43{,}01$

Opgave 8:



In ΔABM geldt: $\angle M = 360° : 8 = 45°$

$\angle A = \angle B = (180° - 45°) : 2 = 67{,}5°$

$\sin 67{,}5° = \dfrac{MN}{6}$

$MN = 6 \cdot \sin 67{,}5° = 5{,}54$

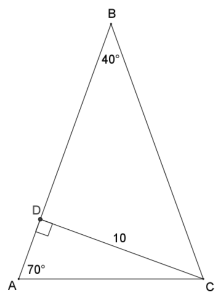
$\cos 67{,}5° = \dfrac{AN}{6}$

$AN = 6 \cdot \cos 67{,}5° = 2{,}30$ dus $AB = 4{,}59$

$Opp(\Delta ABM) = \tfrac{1}{2} \cdot 4{,}59 \cdot 5{,}54 = 12{,}73$

$Opp(ABCDEFGH) = 8 \cdot Opp(\Delta ABM) = 8 \cdot 12{,}73 = 101{,}82$

Opgave 9:



$\tan 70° = \dfrac{CD}{AD} = \dfrac{10}{AD}$

$AD = \dfrac{10}{\tan 70°} = 3{,}64$

$\tan 40° = \dfrac{CD}{BD} = \dfrac{10}{BD}$

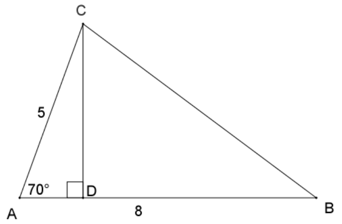
$BD = \dfrac{10}{\tan 40°} = 11{,}92$

$AB = 3{,}64 + 11{,}92 = 15{,}56$

$Opp(\Delta ABC) = \tfrac{1}{2} \cdot AB \cdot CD = \tfrac{1}{2} \cdot 15{,}56 \cdot 10 = 77{,}8$

Opgave 10:

a.



b. $\sin 70° = \frac{CD}{AC} = \frac{CD}{5}$

$CD = 5 \cdot \sin 70° = 4{,}7$

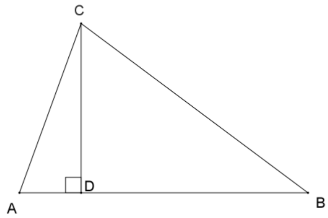
$Opp(\Delta ABC) = \frac{1}{2} \cdot AB \cdot CD = \frac{1}{2} \cdot 8 \cdot 4{,}7 = 18{,}8$

c. $\sin \angle A = \frac{CD}{AC} = \frac{CD}{b}$

$CD = b \cdot \sin \angle A$

$Opp(\Delta ABC) = \frac{1}{2} \cdot AB \cdot CD = \frac{1}{2} \cdot c \cdot b \cdot \sin \angle A$

dus $Opp(\Delta ABC) = \frac{1}{2} \cdot b \cdot c \cdot \sin \angle A$



Opgave 11:

$Opp(cirkelsegment) = \frac{80}{360} \cdot \pi \cdot 10^2 = 69{,}81$

$\sin 40° = \frac{AN}{AM} = \frac{AN}{10}$

$AN = 10 \cdot \sin 40° = 6{,}43$

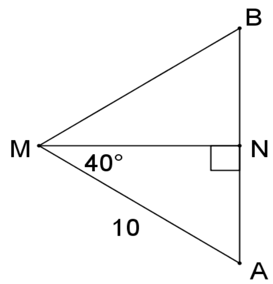
$AB = 2 \cdot 6{,}43 = 12{,}86$

$\cos 40° = \frac{MN}{AM} = \frac{MN}{10}$

$MN = 10 \cdot \cos 40° = 7{,}66$

$Opp(\Delta ABM) = \frac{1}{2} \cdot AB \cdot MN = \frac{1}{2} \cdot 12{,}86 \cdot 7{,}66 = 49{,}24$

$Opp(maantje) = 69{,}81 - 49{,}24 = 20{,}57$



Opgave 12:

$Opp(halve\ cirkel) = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi \cdot 5^2 = 39{,}27$

$Opp(\Delta CDM) = \frac{1}{2} \cdot 5 \cdot 5 \cdot \sin 60° = 10{,}83$

$Opp(ABCDEF) = Opp(halve\ cirkel) + 3 \cdot Opp(\Delta CDM) = 39{,}27 + 3 \cdot 10{,}83 = 71{,}75$

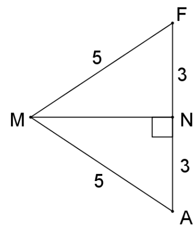
Opgave 13:

$\sin \angle FMN = \frac{FN}{FM} = \frac{3}{5}$

$\angle FMN = 36{,}87°$

$\angle FMA = 73{,}74°$

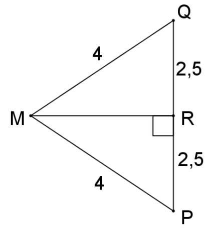
$Opp(\Delta AFM) = \frac{1}{2} \cdot 5 \cdot 5 \cdot \sin 73{,}74° = 12$



$Opp(maantje\ AF) = Opp(cirkelsegment\ AFM) - Opp(\Delta AFM) = \frac{73{,}74}{360} \cdot \pi \cdot 5^2 - 12 = 4{,}088$

$Opp(ABCDEF) = Opp(cirkel) - 3 \cdot Opp(maantje) = \pi \cdot 5^2 - 3 \cdot 4{,}088 = 66{,}3$

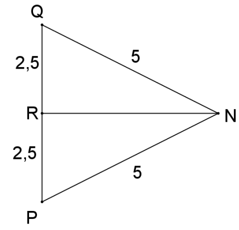
Opgave 14:



$\sin\angle PMR = \frac{PR}{PM} = \frac{2\frac{1}{2}}{4} = 0{,}625$

$\angle PMR = 38{,}68°$

$\angle PMQ = 77{,}36°$



$\sin\angle PNR = \frac{PR}{PN} = \frac{2\frac{1}{2}}{6} = \frac{5}{12}$

$\angle PNR = 24{,}62°$

$\angle PNQ = 49{,}25°$

$Opp(maantje\ PMQ) = Opp(cirkelsegment) - Opp(\Delta PMQ)$

$= \frac{77{,}36}{360} \cdot \pi \cdot 4^2 - \frac{1}{2} \cdot 4 \cdot 4 \cdot \sin 77{,}36° = 2{,}995$

$Opp(maantje\ PNQ) = Opp(cirkelsegment) - Opp(\Delta PNQ)$

$= \frac{49{,}25}{360} \cdot \pi \cdot 6^2 - \frac{1}{2} \cdot 6 \cdot 6 \cdot \sin 49{,}25° = 1{,}836$

$Opp = 2{,}995 + 1{,}836 = 4{,}83$

2.2 Uitslagen

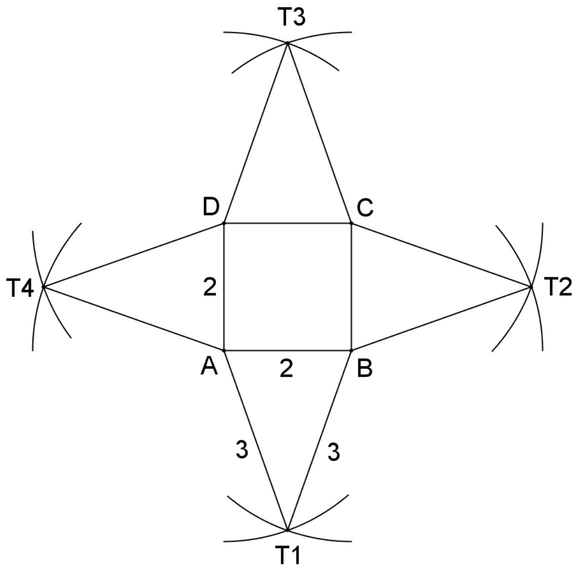
Opgave 15:

figuur a , figuur b en figuur d

Opgave 16:

figuur a , figuur b en figuur d

Opgave 17:



Opgave 18:

a. $\frac{EF}{AB} = \frac{ET}{AT}$

$\frac{2}{4} = \frac{x}{x+3}$

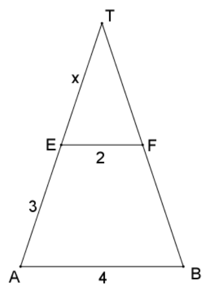
$2(x+3) = 4x$

$2x + 6 = 4x$

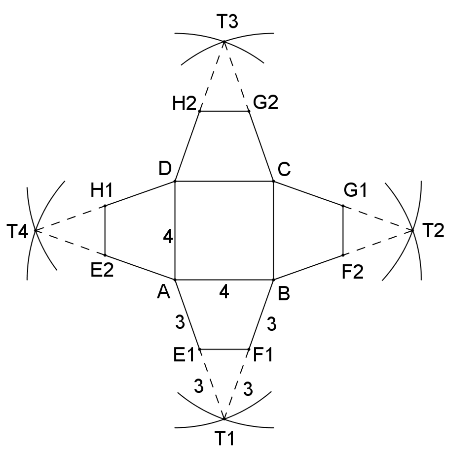
$-2x = -6$

$x = 3$

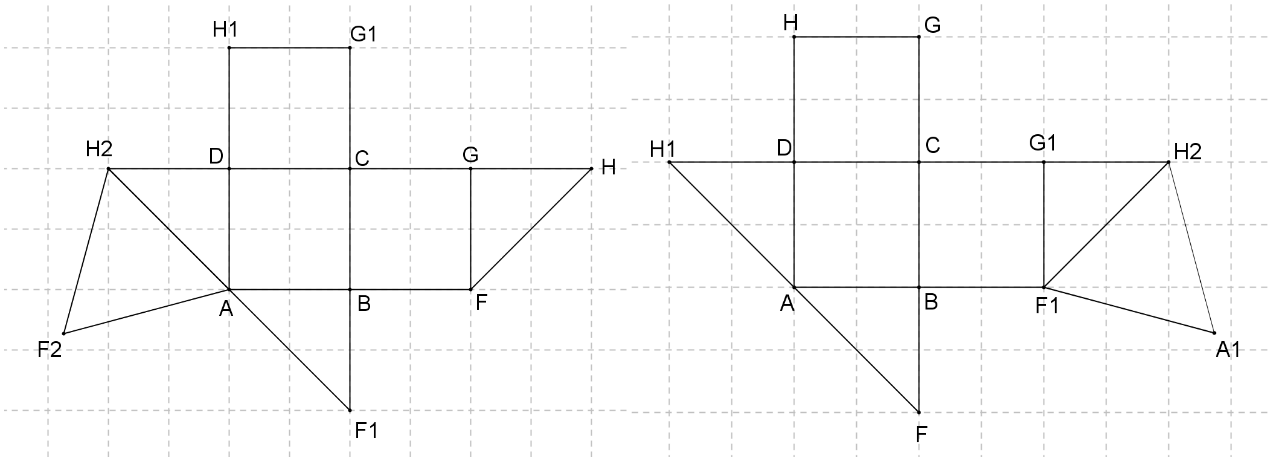
dus $AT = 6$



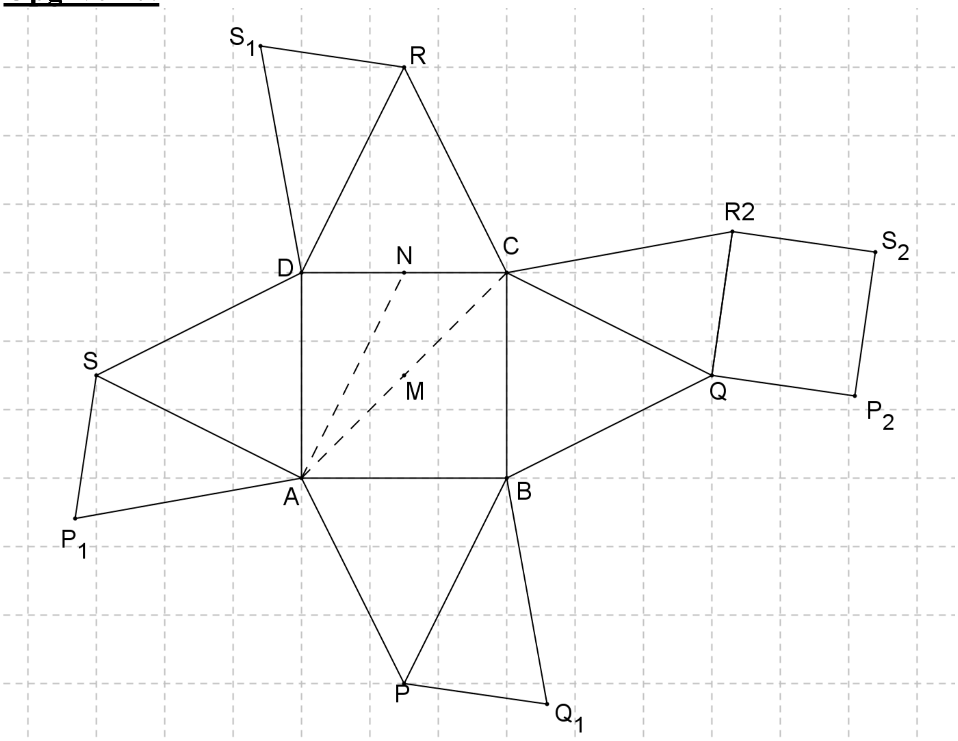
b.



Opgave 19:



Opgave 20:



$AP = BP = BQ = CQ = CR = DR = DS = AS = AN$

$PQ = QR = RS = PS = \frac{1}{2} AC = AM$

Opgave 21:

Figuur b, de lengte van de rechthoek is gelijk aan de omtrek van de cirkel.

Opgave 22:

a. lengte cirkelboog $= \dfrac{90}{360} \cdot 2 \cdot \pi \cdot 3 = 1\frac{1}{2}\pi$

omtrek grondcirkel kegel $= 2\pi r = 1\frac{1}{2}\pi$

$r = \dfrac{1\frac{1}{2}\pi}{2\pi} = \frac{3}{4}$ cm

b. lengte cirkelboog $= \dfrac{210}{360} \cdot 2 \cdot \pi \cdot 2{,}5 = 2\tfrac{11}{12}\pi$

omtrek grondcirkel kegel $= 2\pi r = 2\tfrac{11}{12}\pi$

$r = \dfrac{2\tfrac{11}{12}\pi}{2\pi} = 1\tfrac{11}{24}$ cm

c. lengte cirkelboog $= \dfrac{300}{360} \cdot 2 \cdot \pi \cdot 2 = 3\tfrac{1}{3}\pi$

omtrek grondcirkel kegel $= 2\pi r = 3\tfrac{1}{3}\pi$

$r = \dfrac{3\tfrac{1}{3}\pi}{2\pi} = 1\tfrac{2}{3}$ cm

Opgave 23:

lengte cirkelboog $= \dfrac{p}{360} \cdot 2\pi R = \dfrac{p}{180 \cdot \pi R}$

omtrek cirkelboog $= 2\pi r = \dfrac{p}{180} \cdot \pi R$

$r = \dfrac{p}{360} \cdot R$

Opgave 24:

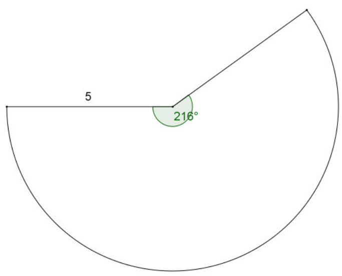
$\dfrac{p}{360} \cdot R = r$

$\dfrac{p}{360} \cdot 5 = 3$

$p = \dfrac{3 \cdot 360}{5} = 216°$

Opgave 25:

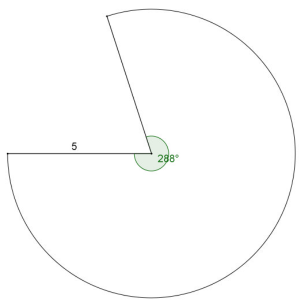
a. zie opgave 24 dus $p = 216°$



b. $R = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$

$\dfrac{p}{360} \cdot 5 = 4$

$p = \dfrac{4 \cdot 360}{5} = 288°$

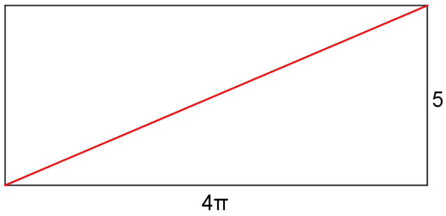


Opgave 26:

a. $\text{lengte} = 2 \cdot \pi \cdot 2 = 4\pi$ cm

$\text{breedte} = 5$ cm

b.



c. $\text{lengte} = \sqrt{(4\pi)^2 + 5^2} = 13{,}5$ cm

Opgave 27:

$r = 4{,}5$ cm

$\text{omtrek vlaggenmast} = 2 \cdot \pi \cdot 4{,}5 = 9\pi$ cm

$\text{lengte touw} = 4 \cdot \sqrt{125^2 + (9\pi)^2} = 513$ cm

2.3 Oppervlakte van ruimtefiguren

Opgave 28:

$Opp(\text{mantel}) = 2\pi rh = 2\cdot\pi\cdot3\cdot4 = 24\pi$

$Opp(cirkel) = \pi r^2 = \pi\cdot3^2 = 9\pi$

$Opp(cilinder) = 2\cdot9\pi + 24\pi = 42\pi = 132$ cm²

Opgave 29:

a. omtrek grondcirkel kegel $= 2\cdot\pi\cdot3 = 6\pi$

$R = \sqrt{4^2+3^2} = \sqrt{25} = 5$

omtrek cirkel met $R = 5$ is $2\cdot\pi\cdot5 = 10\pi$

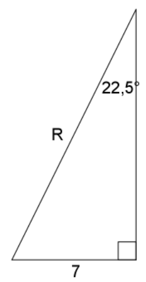
je hebt het $\frac{6\pi}{10\pi} = \frac{3}{5}$ deel

dus $\frac{3}{5}\cdot360° = 216°$

b. $Opp(kegelmantel) = \frac{216}{360}\cdot\pi\cdot5^2 = 47$ cm²

c. $Opp(kegel) = \pi\cdot3^2 + 47 = 75$ cm²

Opgave 30:

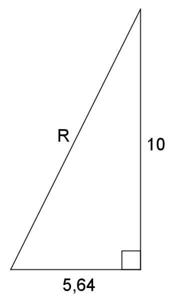


$\sin 22{,}5° = \frac{7}{R}$

$R = \frac{7}{\sin 22{,}5°} = 18{,}29$

$Opp = \pi rR + \pi r^2 = \pi\cdot7\cdot18{,}29 + \pi\cdot7^2 = 556{,}2$ cm²

Opgave 31:



$Opp(grondcirkel) = \pi r^2 = 100$

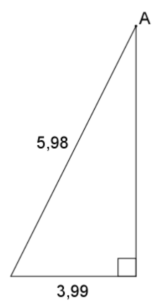
$r^2 = \frac{100}{\pi}$

$r = \sqrt{\frac{100}{\pi}} = 5{,}64$

$R = \sqrt{10^2 + 5{,}64^2} = 11{,}48$

$Opp(kegelmantel) = \pi rR = \pi\cdot5{,}64\cdot11{,}48 = 203{,}4$ cm²

Opgave 32:



$Opp(grondcirkel) = \pi r^2 = 50$

$r^2 = \frac{50}{\pi}$

$r = \sqrt{\frac{50}{\pi}} = 3{,}99$

$Opp(kegelmantel) = \pi rR = \pi\cdot3{,}99\cdot R = 75$

$R = 5{,}98$

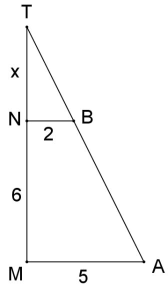
$\sin\angle A = \frac{3{,}99}{5{,}98}$

$\angle A = 41{,}85°$

tophoek $= 84°$

Opgave 33:

a. $\frac{TN}{TM} = \frac{BN}{AM}$



$\frac{x}{x+6} = \frac{2}{5}$

$5x = 2(x+6)$

$5x = 2x + 12$

$3x = 12$

$x = 4$

b. $R = \sqrt{10^2 + 5^2} = \sqrt{125}$

$Opp(kegelmantel) = \pi r R = \pi \cdot 5 \cdot \sqrt{125} = 175{,}62$

c. bovenste kegel:

$R = \sqrt{4^2 + 2^2} = \sqrt{20}$

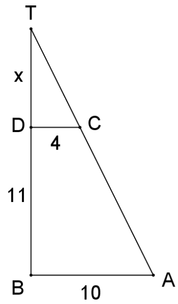
$Opp(kegelmantel) = \pi r R = \pi \cdot 2 \cdot \sqrt{20} = 28{,}10$

$Opp(\text{ mantel afgeknotte kegel }) = 175{,}62 - 28{,}10 = 147{,}52$

d. $Opp = 147{,}52 + \pi \cdot 2^2 + \pi \cdot 5^2 = 238{,}63$

Opgave 34:

$\frac{DT}{BT} = \frac{CD}{AB}$



$\frac{x}{x+4} = \frac{4}{10}$

$10x = 4(x+11)$

$10x = 4x + 44$

$6x = 44$

$x = \frac{44}{6} = 7\frac{1}{3}$

$AT = \sqrt{10^2 + (18\frac{1}{3})^2} = 20{,}88$

$CT = \sqrt{4^2 + (7\frac{1}{3})^2} = 8{,}35$

$Opp = \pi \cdot 10 \cdot 20{,}88 - \pi \cdot 4 \cdot 8{,}35 = 551\,\text{cm}^2$

Opgave 35:

$Opp(I) = 2 \cdot \pi \cdot 4^2 + 2 \cdot \pi \cdot 4 \cdot 10 = 112\pi$

$Opp(II) = 2 \cdot o \cdot 2^2 + 2 \cdot \pi \cdot 2 \cdot h = 8\pi + 4\pi h$

$8\pi + 4\pi h = 112\pi$

$4\pi h = 104\pi$

$h = 26$

Opgave 36:

$Opp(\text{ halve cilinder }) = \frac{1}{2} \cdot 2 \cdot \pi \cdot 3 \cdot 6 = 18\pi$

$Opp(rechthoek) = 6 \cdot 6 = 36$

$Opp = 18\pi + 36$

$K = (18\pi + 36) \cdot 175 = 16196$ euro

Opgave 37:

$Opp = \frac{1}{2} \cdot Opp(cilindermantel) = \frac{1}{2} \cdot 2 \cdot \pi \cdot 12 \cdot 20 = 754$ cm²

Opgave 38:

$2 \cdot 4 \cdot r^2 = 4 \cdot \pi \cdot 5^2$

$8\pi r^2 = 100\pi$

$r^2 = 12{,}5$

$r = 3{,}54$

Opgave 39:

$Omtrek = 2\pi R = 40000$

$R = 6366$

$Opp = 4\pi r^2 = 4 \cdot \pi \cdot 6366^2 = 509295818$

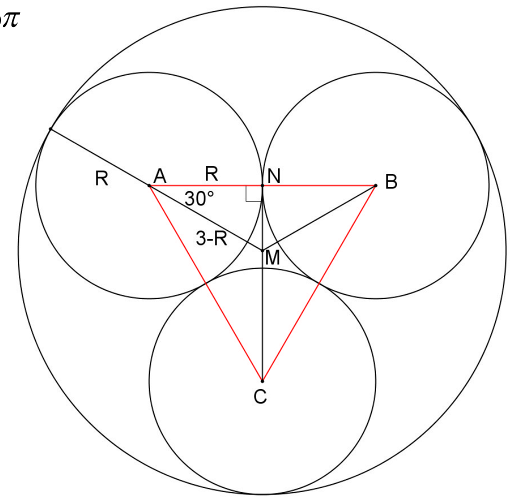
$0{,}71 \cdot 509295818 = 361600031$ km²

Opgave 40:

a. $Opp(bal) = 4\pi r^2 = 4 \cdot \pi \cdot 3^2 = 36\pi$

$Opp(\text{halve cilindermantel}) = \frac{1}{2} \cdot 2\pi rh = \pi \cdot 3 \cdot 12 = 36\pi$

dus Linda heeft gelijk



b. in ΔAMN geldt:

$\cos 30° = \dfrac{R}{3-R}$

$0{,}866 = \dfrac{R}{3-R}$

$0{,}866 \cdot (3-R) = R$

$2{,}598 - 0{,}866R = R$

$-1{,}866R = -2{,}598$

$R = 1{,}39$

c. de hoogte van een laag knikkers is: $2 \cdot 1{,}39 = 2{,}78$ cm

de hoogte van vier lagen knikkers is: $4 \cdot 2{,}78 = 11{,}12 < 12$

d. $Opp = 12 \cdot 4 \cdot \pi \cdot 1{,}39^2 = 291$ cm²

e. $291 : (2 \cdot 36\pi) = 1{,}29 \times$ zo groot

2.4 Inhoud van ruimtefiguren

Opgave 41:

a. $Inh(kegel) = \frac{1}{3}\pi r^2 h$

b. $Inh(bol) = 4 \cdot \frac{1}{3}\pi r^2 h = \frac{4}{3}\pi r^2 h$ maar voor de bol geldt: $h = r$

$Inh(bol) = \frac{4}{3}\pi r^3$

Opgave 42:

neem voor de straal van de tennisbal r , dan is $h_{cilinder} = 6r$

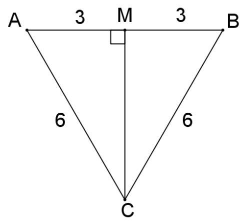
$Inh(cilinder) = \pi r^2 h = \pi r^2 \cdot 6r = 6\pi r^3$

$Inh(tennisbal) = \frac{4}{3}\pi r^3$

$Inh(3\ tennisballen) = 3 \cdot \frac{4}{3}\pi r^3 = 4\pi r^3$

dus $\frac{4\pi r^3}{6\pi r^3} \cdot 100\% = 66{,}7\%$

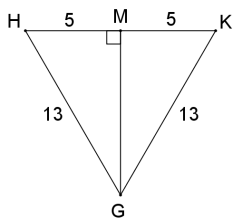
Opgave 43:



a. $CM = \sqrt{6^2 - 3^2} = \sqrt{27}$

$Opp(\Delta ABC) = \frac{1}{2} \cdot 6 \cdot \sqrt{27} = 3\sqrt{27}$

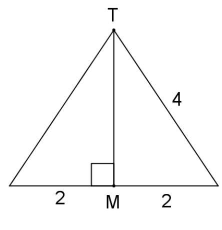
$Inh(ABC\,DEF) = G \cdot h = 3\sqrt{27} \cdot 8 = 124{,}7$



b. $GM = \sqrt{13^2 - 5^2} = \sqrt{144} = 12$

$Opp(\Delta GHK) = \frac{1}{2} \cdot 10 \cdot 12 = 60$

$Inh(L\,GHK) = \frac{1}{3} \cdot G \cdot h = \frac{1}{3} \cdot 60 \cdot 9 = 180$



c. $TM = \sqrt{4^2 - 2^2} = \sqrt{12}$

$Inh(kegel) = \frac{1}{3} \cdot G \cdot h = \frac{1}{3} \cdot \pi \cdot 2^2 \cdot \sqrt{12} = 14{,}5$

Opgave 44:

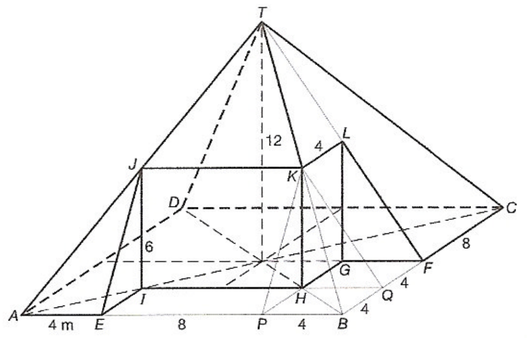
$h = \sqrt{2^2 - 1^2} = \sqrt{3}$

$Opp(G) = 3{,}5$

$Inh = \frac{1}{3} \cdot G \cdot h = \frac{1}{3} \cdot 3{,}5 \cdot \sqrt{3} = 2{,}0\,\text{cm}^3$

Opgave 45:

$Inh(T\ \ ABCD) = \frac{1}{3} \cdot G \cdot h = \frac{1}{3} \cdot 16^2 \cdot 12 = 1024$



voorste prisma:

$Opp(\Delta EIJ) = \frac{1}{2} \cdot 4 \cdot 6 = 12$

$Inh(EIJ\ \ PHK) = G \cdot h = 12 \cdot HI = 12 \cdot 8 = 96$

prisma rechts:

$Opp(\Delta FGL) = \frac{1}{2} \cdot 4 \cdot 6 = 12$

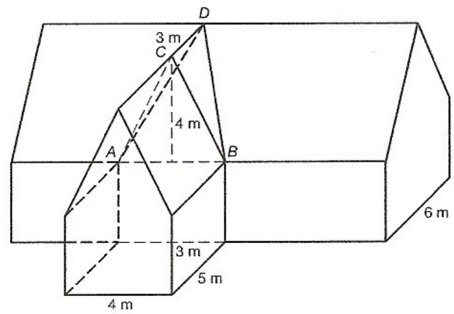
$Inh(FGL\ QHK) = G \cdot h = 12 \cdot GH = 12 \cdot 4 = 48$

kleine piramide

$Inh(K\ \ BPHQ) = \frac{1}{3} \cdot G \cdot h = \frac{1}{3} \cdot 4^2 \cdot 6 = 32$

$Inh(woning) = 1024 - 96 - 48 - 32 = 848\,\text{m}^3$

Opgave 46:



$Opp(\Delta ABC) = \frac{1}{2} \cdot 4 \cdot 4 = 8$

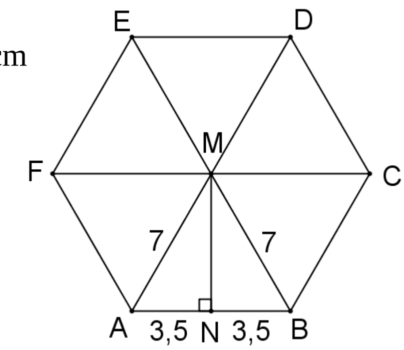
$Inh(D\ \ ABC) = \frac{1}{3} \cdot 8 \cdot 3 = 8$

Je kunt het huis splitsen in aan de onderkant twee balken en boven twee keer een prisma en een piramide.

$Inh(huis) = 4 \cdot 5 \cdot 3 + 14 \cdot 6 \cdot 3 + \frac{1}{2} \cdot 4 \cdot 4 \cdot 5 + 8 + \frac{1}{2} \cdot 6 \cdot 4 \cdot 14 = 528\,\text{m}^3$

Opgave 47:

het grondvlak is een regelmatige zeshoek met zijde $5 \cdot 1{,}4 = 7$ cm



$h = 5 \cdot 2{,}75 = 13{,}75 = \text{cm}$

$MN = \sqrt{7^2 - 3{,}5^2} = \sqrt{36{,}75}$

$Opp(\Delta ABM) = \frac{1}{2} \cdot 7 \cdot \sqrt{36{,}75} = 3{,}5\sqrt{36{,}75}$

$Opp(zeshoek) = 6 \cdot 3{,}5\sqrt{36{,}75}$

$Inh = G \cdot h = 21\sqrt{36{,}75} \cdot 13{,}75 = 1750\,\text{cm}^3$

Opgave 48:

$Inh(bol) = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \cdot 5^3 = \frac{500}{3}\pi$

$Inh(kegel) = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \cdot r^2 \cdot 10 = \frac{500}{3}\pi$

$r^2 = 50$

$r = \sqrt{50} = 7{,}1\,\text{cm}$

$Inh(cilinder) = \pi r^2 h = \pi \cdot r^2 \cdot 10 = \frac{500}{3}\pi$

$r^2 = \frac{50}{3}$

$r = \sqrt{\frac{50}{3}} = 4{,}1\,\text{cm}$

Opgave 49:

$Inh(kegel) = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \cdot r^2 \cdot 2r = \frac{2}{3}\pi r^3$

$Inh(bol) = \frac{4}{3}\pi r^3$

$Inh(cilinder) = \pi r^2 h = \pi r^2 \cdot 2r = 2\pi r^3$

$Inh(kegel) : Inh(bol) : Inh(cilinder) = \frac{2}{3}\pi r^3 : \frac{4}{3}\pi r^3 : 2\pi r^3 = 1 : 2 : 3$

Opgave 50:

a. $Inh(balk) = 15 \cdot 15 \cdot 200 = 45000$ cm³

$Inh(cilinder) = 4 \cdot \pi r^2 h = 4 \cdot \pi \cdot 7{,}5^2 \cdot 92{,}5 = 65384$ cm³

$Inh(totaal) = 45000 + 65384 = 110384$ cm³

b. $\pi \cdot 7{,}5^2 \cdot h = 110384$

$h = 625$ cm

Opgave 51:

a. $AC = \sqrt{AB^2 + BC^2} = \sqrt{4^2 + 4^2} = \sqrt{32}$

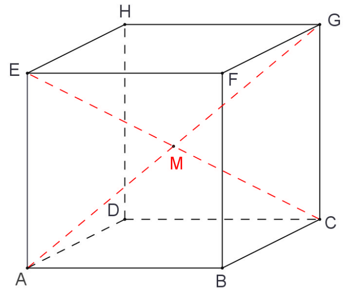
$AG = \sqrt{AC^2 + CG^2} = \sqrt{(\sqrt{32})^2 + 4^2} = \sqrt{48}$

$AM = \frac{1}{2} AG = \frac{1}{2}\sqrt{48}$

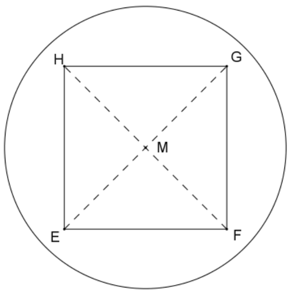
$r = \frac{1}{2}\sqrt{48}$

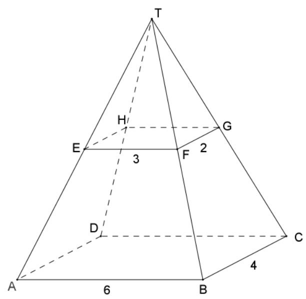
$Inh = Inh(bol) - Inh(kubus)$

$= \frac{4}{3}\pi \cdot (\frac{1}{2}\sqrt{48})^3 - 4^3 = 110{,}12$ cm³



b.





Opgave 52:

a. de voorzijde is een snavelfiguur met vergrotingsfactor 2
de rechterzijkant is een snavelfiguur met vergrotingsfactor 2

b. $h = 4$

$Inh(karretje) = \frac{1}{3} \cdot 3 \cdot 2 \cdot 4 - \frac{1}{3} \cdot 1{,}5 \cdot 1 \cdot 2 = 7$ m³

Opgave 53:

neem alle afmetingen in dm

$\frac{x}{x + 2{,}5} = \frac{1}{1{,}5}$

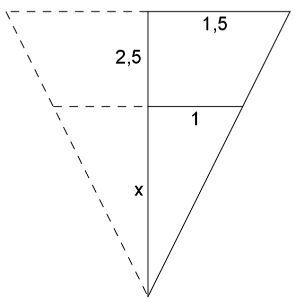
$1{,}5x = x + 2{,}5$

$0{,}5x = 2{,}5$

$x = 5$

$Inh = \frac{1}{3}\pi \cdot 1{,}5^2 \cdot 7{,}5 - \frac{1}{3}\pi \cdot 1^2 \cdot 5 = 12{,}4$ dm³

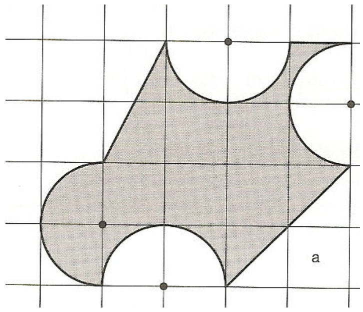
dus de inhoud van de emmer is 12,4 liter



2.5 Diagnostische toets

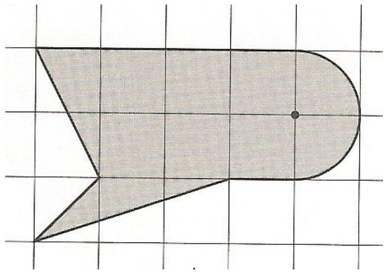
Opgave 1:

a. $Opp = 4^2 - \frac{1}{2}\cdot 2\cdot 1 - \frac{1}{2}\cdot 2\cdot 2 - \pi\cdot 1^2 = 9{,}86\text{ cm}^2 = 986\text{mm}^2$



b. $Opp = 4\cdot 3 - 1^2 - \frac{1}{2}\cdot 3\cdot 1 - \frac{1}{2}\cdot 1\cdot 1 - \frac{1}{2}\cdot 2\cdot 1 + \frac{1}{2}\pi\cdot 1^2$

$= 9{,}57\text{ cm}^2 = 957\text{mm}^2$

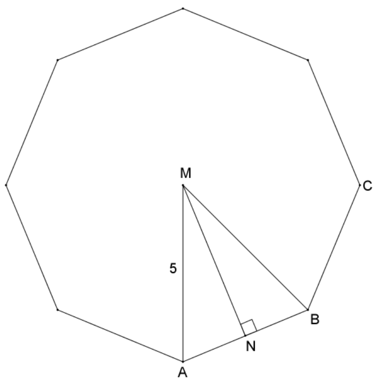


Opgave 2:

a. $\angle AMB = \frac{360}{8} = 45°$

$\angle ABM = \frac{180-45}{2} = 67{,}5°$

$\angle ABC = 2\cdot 67{,}5° = 135°$



b. $Omtrek\ cirkel = 2\cdot\pi\cdot r = 10\pi$

$r = 5$ dus $Am = 5$

$\cos 67{,}5° = \frac{AN}{5}$

$AN = 5\cos 67{,}5° = 1{,}91$

$AB = 2\cdot AN = 3{,}83$

$Omtrek(ABCDEFGH) = 8\cdot AB = 30{,}6$

$\sin 67{,}5° = \frac{MN}{5}$

$MN = 5\sin 67{,}5° = 4{,}62$

$Opp(\Delta ABM) = \frac{1}{2}\cdot AB\cdot MN = \frac{1}{2}\cdot 3{,}83\cdot 4{,}62 = 8{,}84$

$Opp(ABCDEFGH) = 8\cdot Opp(\Delta ABM) = 70{,}7$

Opgave 3:

$MN = \sqrt{5^2 - 3^2} = \sqrt{16} = 4$

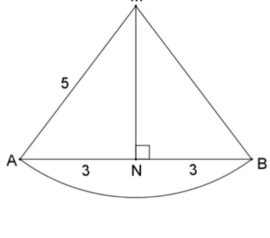
$Opp(\Delta ABM) = \frac{1}{2}\cdot AB\cdot MN = \frac{1}{2}\cdot 6\cdot 4 = 12$

$\sin\angle AMN = \frac{3}{5}$

$\angle AMN = 36{,}9°$ dus $\angle AMB = 2\cdot 36{,}9 = 73{,}7°$

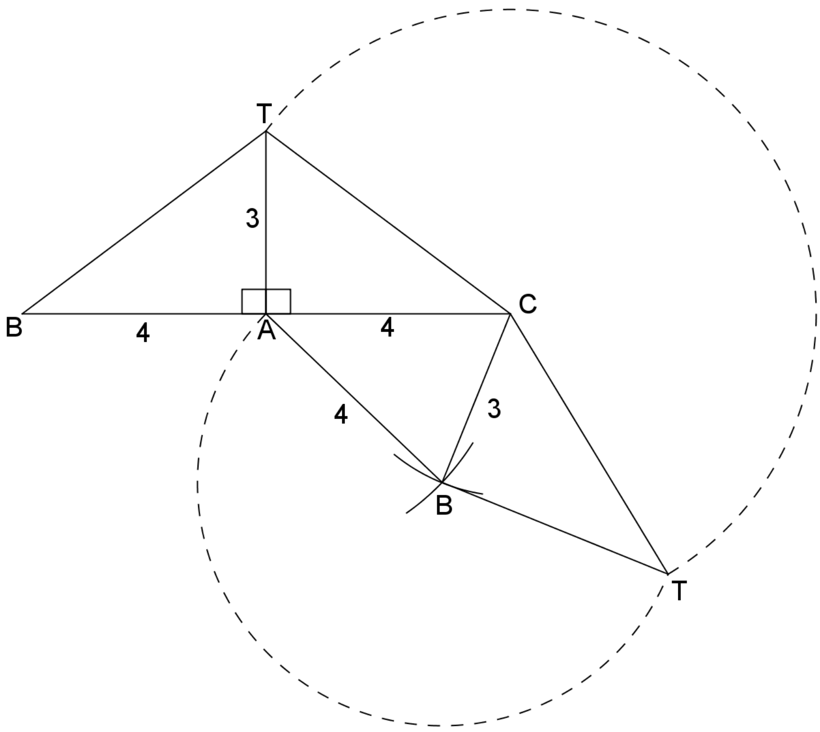
$Opp(cirkel\sec tor) = \frac{73{,}7}{360}\cdot\pi\cdot 5^2 = 16{,}09$

$Opp(rode\ segment) = 16{,}09 - 12 = 4{,}09$

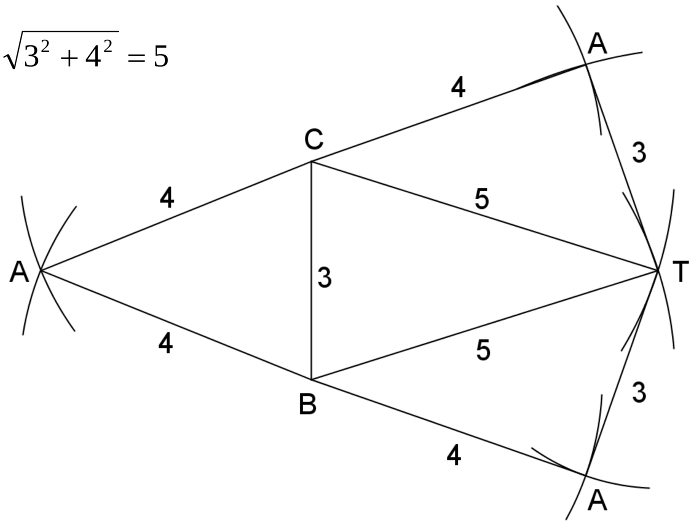


Opgave 4:

a.



b. $BT = CT = \sqrt{3^2 + 4^2} = 5$



Opgave 5:

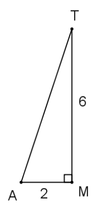
a. $AT = \sqrt{6^2 + 2^2} = \sqrt{40}$

$omtrek\ grondcirkel = 2 \cdot \pi \cdot 2 = 4\pi$

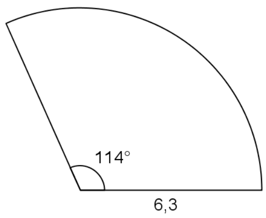
$omtrek\ cirkel = 2\pi\sqrt{40}$

dus je hebt het $\frac{4\pi}{2\pi\sqrt{40}} = \frac{2}{\sqrt{40}}$ deel van de cirkel

$middelpuntshoek = \frac{2}{\sqrt{40}} \cdot 360 = 113{,}8°$



b. $\sqrt{40} = 6{,}3$



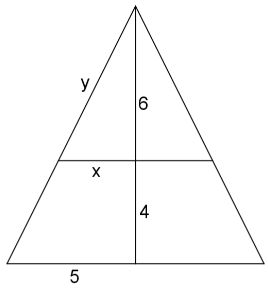
Opgave 6:

a. $R = \sqrt{10^2 + 5^2} = \sqrt{125}$

$Opp(kegelmantel) = \pi \cdot r \cdot R = \pi \cdot 5 \cdot \sqrt{125} = 175{,}62$

$Opp(grondcirkel) = \pi\, r^2 = \pi \cdot 5^2 = 78{,}54$

$Opp(kegel) = 175{,}62 + 78{,}54 = 254{,}2$



b. $\frac{x}{5} = \frac{6}{10} = \frac{y}{\sqrt{125}}$

$x = 3 \quad y = \frac{6}{10}\sqrt{125}$

$Opp = \pi r R - \pi x y = \pi \cdot 5 \cdot \sqrt{125} - \pi \cdot 3 \cdot \frac{6}{10}\sqrt{125} = 112{,}4$

Opgave 7:

a. $Opp(cilinder) = 2 \cdot \pi r^2 + 2\pi r h = 2 \cdot \pi \cdot 4^2 + 2 \cdot \pi \cdot 4 \cdot 8 = 32\pi + 64\pi = 96\pi$

$Opp(bol) = 4\pi r^2 = 4 \cdot \pi \cdot 4^2 = 64\pi$

$\frac{96\pi}{64\pi} \cdot 100\% = 150\%$ dus 50% meer

b. $Opp(cilinder) = 2 \cdot \pi r^2 + 2\pi r h = 2 \cdot \pi \cdot r^2 + 2 \cdot \pi \cdot r \cdot 2r = 2\pi r^2 + 4\pi r^2 = 6\pi r^2$

$Opp(bol) = 4\pi r^2$

$\frac{6\pi r^2}{4\pi r^2} \cdot 100\% = 150\%$ dus 50% meer

dus het percentage hangt niet af van de straal van de bol

Opgave 8:

a. $r = 5$ en $h = 10$

$Inh = \pi r^2 h = \pi \cdot 5^2 \cdot 10 = 785$ cm³

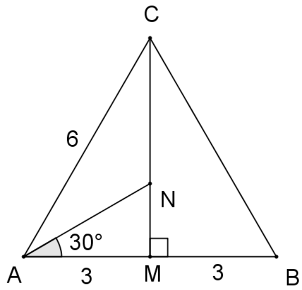
b. $r = 5$ dus $Inh(bol) = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \cdot 5^3 = 524$ cm³

Opgave 9:

a. $CM = \sqrt{6^2 - 3^2} = \sqrt{27}$

$Opp(\Delta ABC) = \frac{1}{2} \cdot AB \cdot CM = \frac{1}{2} \cdot 6 \cdot \sqrt{27} = 3\sqrt{27}$

$Inh = \frac{1}{3} \cdot G \cdot h = \frac{1}{3} \cdot 3\sqrt{27} \cdot 10 = 51{,}96$



b. de hoogte wordt $0{,}6\times$zo groot, dus iedere zijde wordt $0{,}6\times$zo groot, dus de inhoud wordt $0{,}6^3 = 0{,}216\times$zo groot

$Inh = 51{,}96 - 0{,}216 \cdot 51{,}96 = 40{,}74$

c. $\cos 30° = \frac{3}{AN}$

$r = AN = 3\cos 30° = 3{,}464$

$Inh = \frac{1}{3}\pi r^2 h - \frac{1}{3} \cdot G \cdot h = \frac{1}{3}\pi \cdot 3{,}464^2 \cdot 10 - 51{,}96 = 73{,}70$